BARRA

Predicted Beta

BARRA Predicted Beta

Beta is a gauge of the expected response of a stock, bond, or portfolio to the overall market. For example, a stock with a beta of 1.5 has an expected excess return of 1.5 times the market excess return. If the market is up 10% over the risk-free rate, then—other things held equal—the portfolio is expected to be up 15%. Beta is one of the most significant means of measuring portfolio risk and shows a strong relationship to expected return.

Historical Beta vs. Predicted Beta

Historical beta is calculated after the fact by running a regression (often over 60 months) on a stock's excess returns against the market's excess returns. There are two important problems with this simple historical approach:

- It does not recognize fundamental changes in the company's operations. For example, when RJR Nabisco spun off its tobacco holdings in 1999, the company's risk characteristics changed significantly. Historical beta would recognize this change only slowly, over time.

- It is influenced by events specific to the company that are unlikely to be repeated. For example, the December 1984 Union Carbide accident in Bhopal, India, took place in a bull market, causing the company's historical beta to be artificially low.

Predicted beta, the beta BARRA derives from its risk model, is a forecast of a stock's sensitivity to the market. It is also known as *fundamental beta*, because it is derived from fundamental risk factors. In the BARRA model these risk factors include 13 attributes—such as size, yield, and price/earnings ratio—plus industry exposure allocated across a maximum of 6 of 55 industry groups. Because we reestimate these risk factors monthly, the predicted beta reflects changes in the company's underlying risk structure in a timely manner.

BARRA programs use predicted beta rather than historical beta because it is a better forecast of market sensitivity.

Computing Predicted Beta

Below we show how the predicted beta of a portfolio is computed.

The beta of a portfolio *p* with respect to the market *m* is defined as the covariance of the portfolio return with the market return divided by the variance of the market:

$$(1) \quad \beta_p = \frac{\mathrm{COV}(r_p, r_m)}{\mathrm{VAR}_m}$$

The covariance between two portfolios is decomposed into two parts: a) the part explained by factors, called *common factor covariance*; and b) the part unexplained by factors, called *specific covariance*.

The factor covariance between portfolio *p* and the return on the market *m* is the product of the transposed vector of the factor exposures for the portfolio, the factor covariance matrix, and the vector of the factor exposures for the market:

$$(2) \quad \mathrm{CF\ COV}(r_p, r_m) = X_p^T \; F \; X_m$$

The specific covariance is:

$$(3) \quad \mathrm{SP\ COV}(r_p, r_m) = \sum_{i=1}^{N} h_{pi} h_{mi} \sigma_i^2$$

Now, combining equations (1) and

$$(4) \quad \mathrm{COV}(r,r) = \mathrm{VAR}(r)$$

we have the formula for the BARRA predicted beta of a portfolio:

$$(5) \quad \beta_p = \frac{\mathrm{COV}(r_p, r_m)}{\mathrm{VAR}_m}$$

$$= \frac{\mathrm{CF\ COV}(r_p, r_m) + \mathrm{SP\ COV}(r_p, r_m)}{\mathrm{CF\ COV}(r_m, r_m) + \mathrm{SP\ COV}(r_m, r_m)}$$

$$= \frac{\sum_{j=1}^{NFAC} \sum_{k=1}^{NFAC} X_{pj} Fjk \, X_{mk} + \sum_{i=1}^{N} h_{pi} h_{mi} \sigma_i^2}{\sum_{j=1}^{NFAC} \sum_{k=1}^{NFAC} X_{mj} Fjk \, X_{mk} + \sum_{i=1}^{N} h_{mi}^2 \sigma_i^2}$$

where

NFAC	is the number of factors (68 in U.S. E2)
N	is the number of assets in the market portfolio
X_{pj}	is the portfolio's exposure to factor *j*
F_{jk}	is the covariance between factors *k* and *j*
X_{mj}	is the market's exposure to factor *j*
h_{pi}	is the holding of the portfolio in asset *i*
h_{mi}	is the holding of the market in asset *i*
σ_i^2	is the specific variance of asset *i*
VAR_m	is the variance of the market